National Aeronautics and Space Administration

Information Theory Applied to Decision Making Structures

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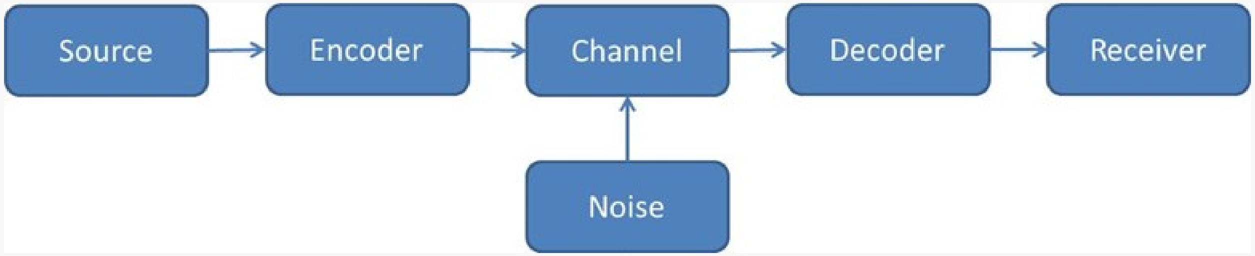
Space Launch System

System Information Flow in the Development Organization

- **System Information flows through the organization and into the system design**
- **Design options driven by technical, budget, schedule, policy and law considerations flow through decision boards**
- **The Information Flow is described by Information Theory**
  - The decision board actions are also described by Information Theory
- **Communication System Model**



Information Representation

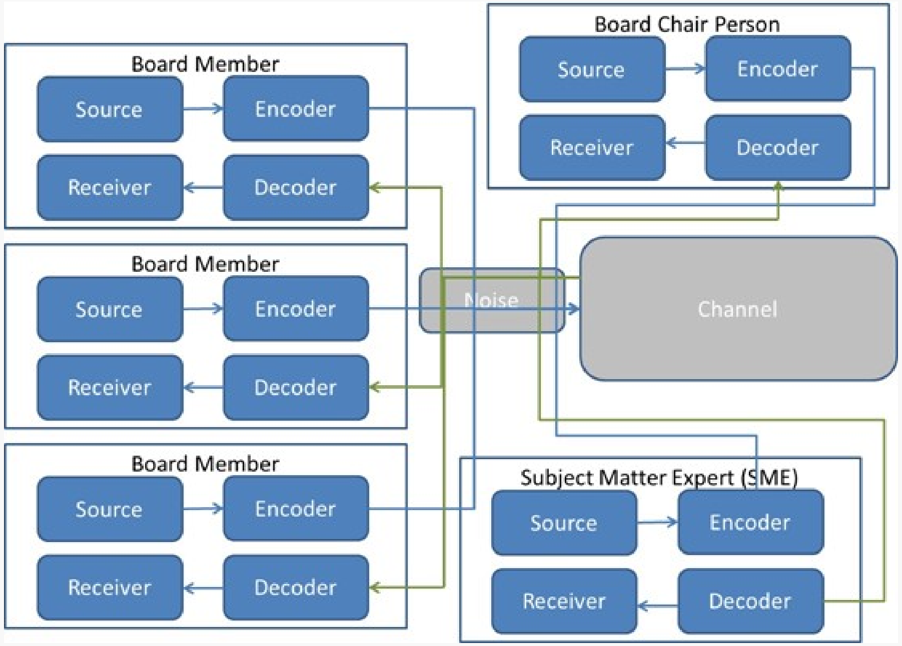
Information

- $I = -\log p_n$

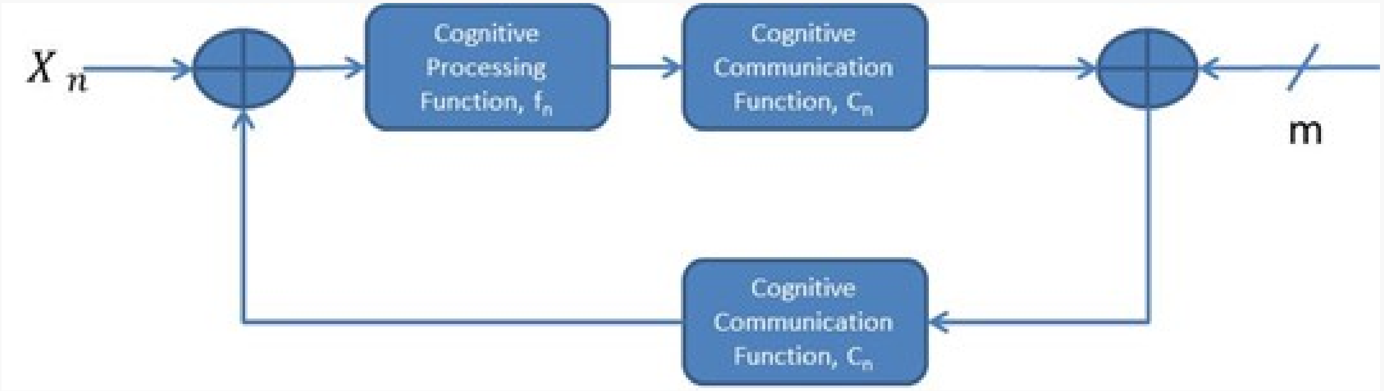
Information Entropy (Uncertainty)

- $\bar{I} = H = -\sum_n p_n \log p_n$

- If information full understood, there is no uncertainty
  - $P_n = 1 \Rightarrow H = 0$



Cognitive Model

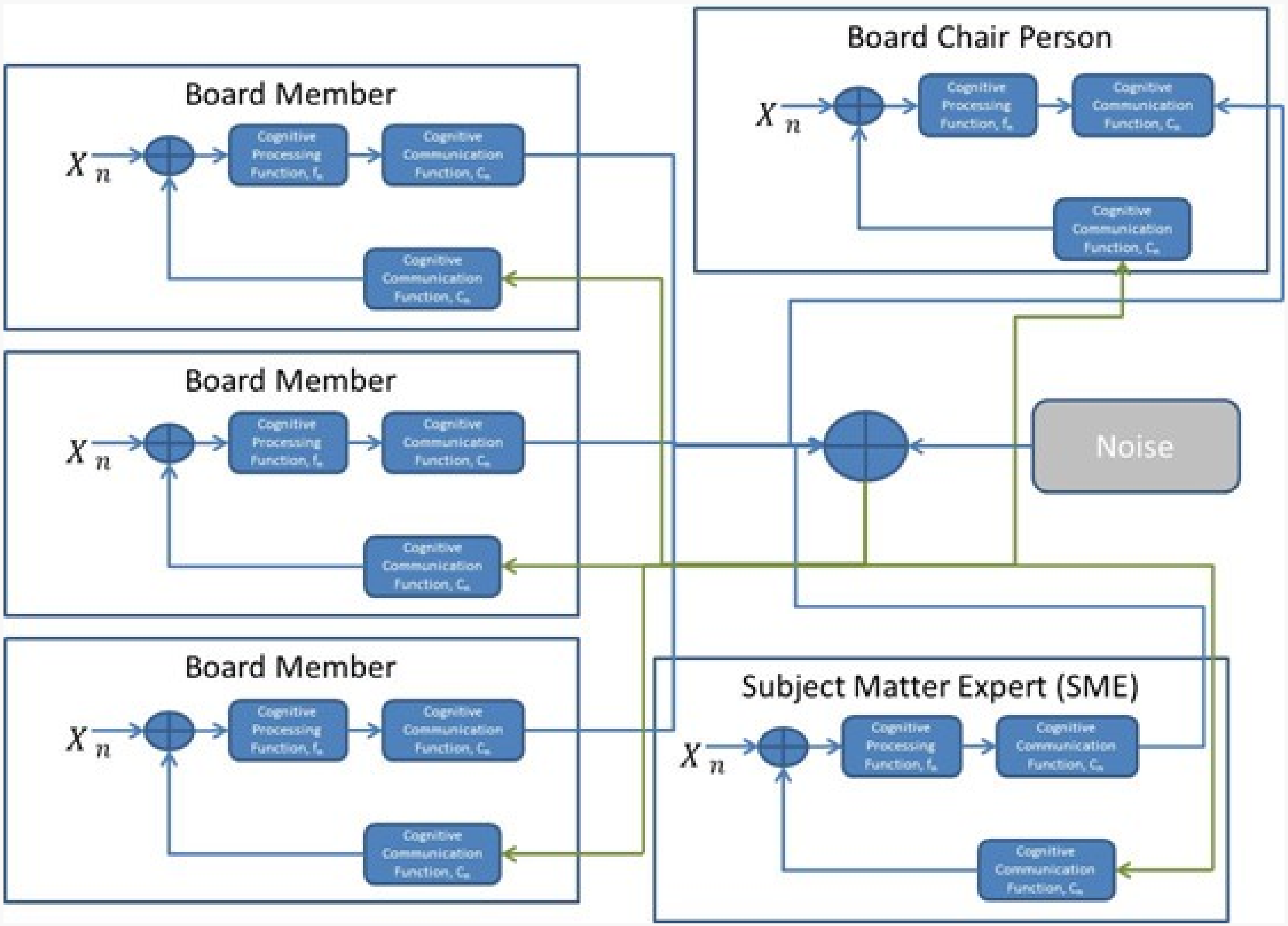


- ***Control Theory and Cognitive Theory can be applied to develop the Board Member relationships and feedback needed for Information Theory flows***

- $Y_n = X_n + \sum_q f_{p,q+1}\left\{C_p\left[f_{p,q}(X_n) + \sum_{m.m\neq p} C_m[f_m(X_n)] + Noise\right]\right\}$

- $T_n = \frac{X_n + f_{p,q}(X_n)}{X_n + \sum_q f_{p,q+1}\left\{C_p\left[f_{p,q}(X_n) + \sum_{m.m\neq p} C_m[f_m(X_n)] + Noise\right]\right\}}$

Cognitive Model



Multiple Board Structures

- **Information Theory provides guidelines on structuring multiple boards**
  - Delegation leads to increased uncertainty
    - $H(S,D,X,Y,Z) \leq H(S) + H(D) + H(X) + H(Y) + H(Z)$
      - Decision should stay at the largest common denominator
  - Overlapping roles for Board members leads to increased uncertainty
    - $H(p_1, p_2, \ldots, p_n, q_1, q_2, \ldots q_m) \geq H(p_1, p_2, \ldots, p_n)$
  - Board structure can have delegated boards when scopes do not overlap
    - $I_A \not\subset I_B$ and $I_B \not\subset I_A$
  - Boards cannot be delegated when there is overlapping scope
    - $I_A \cap I_B$
    - $I_A \subset I_B$ and/or $I_B \subset I_A$

Statistical Properties of Boards

Continuity

- $H(p_1, p_2, \ldots, p_n)$ is continuous in all p_n. Thus, there are no discontinuities in the information probabilities. This means, as noted earlier, that the range maps completely to the domain within the board. Discontinuities lead to highly uncertain, or in some cases blind, decisions. A robust board has all disciplines (i.e., affected or contributing parties) represented. This satisfies the range to domain mapping criteria and the related Continuity property.

Symmetry

- $H(p_1, p_2, \ldots, p_n) = H(p_2, p_1, \ldots, p_n)$. Thus, the order of uncertainty does not contribute to the uncertainty in the decisions. This must be distinguished from temporal order of information sharing leading to a momentary information void on a subject until all aspects are explained for understanding. The process of understanding is always assumed to be complete in this model, and symmetry holds for a complete understanding of a subject. The order in which you discuss or think of a subject does not matter if you fully understand the subject.

Extrema

- $\boldsymbol{Max[H(p_1, p_2, \ldots, p_n)] = H(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})}$. The maximum uncertainty arrives when all decisions are equally uncertain. If any single decision can be distinguished from the others, then the uncertainty to choose or not choose that option is smaller. Similarly, if no options satisfy the decision criteria, then the board has no information on which to base a decision leading to
- $\boldsymbol{Min[H(p_1, p_2, \ldots, p_n)] = H(0, 0, \ldots, 0) = 0}$.

Additivity

- If a probability of occurrence, p_n, can be subdivided into smaller segments, q_k, then the uncertainty can be represented as
- $H\left(p_1, p_2, \ldots, p_{n-1}, q_1, q_2, \ldots q_k\right) = H\left(p_1, p_2, \ldots, p_n\right) + p_n H\left(\frac{q_1}{p_n}, \frac{q_2}{p_n}, \ldots \frac{q_k}{p_n}\right).$

Application Principles

1. **Uncertainty exists in complex decisions.**
   - In these cases, simplifying assumptions lead to a lower understanding of the decision intricacies and a higher uncertainty (not always recognized) in the decision process. Interactions among differing factors in complex decisions have dependencies that are not recognized (ignorance)iv or not well understood. Missing information is not always easily recognized. Factors not considered important in the decision can end up driving the system. Missing information comes from events (physical, chronological, or fiscal) not recognized as relating to the decision, unknown environments in which a system operates, unrecognized dependencies, and cultural biases (e.g., politics).

2. **The uncertainty of which option is best collectively, and in some cases individually, leads to a statistical representation of which answer is best.**
   - In a board decision, the board vote is a statistical event with a distribution of yes and no positions. This is tied back to the cognitive functions. This statistical function is then combined with other statistical functions (i.e., other board members and SMEs) to produce a decision based on these functions.

3. **The potential for misunderstanding (i.e., error) is also statistical.**
   - This includes miscommunication (not stating clearly what is meant or not understanding clearly what is stated (and therefore meant). These lead to unintended consequences in the decision-making process. These unintended consequences can be social, physical, chronological, fiscal, or environmental.

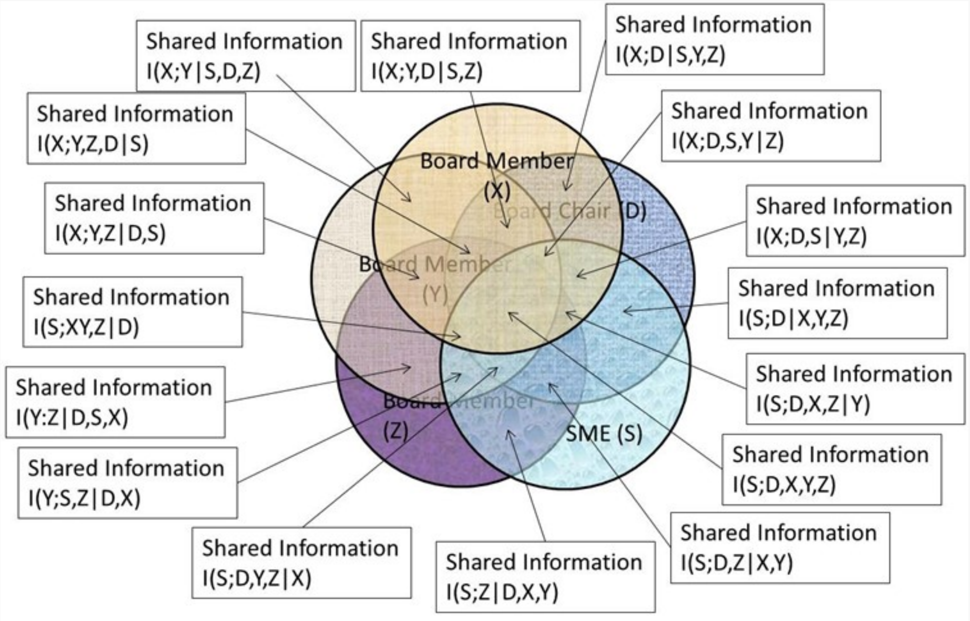
4. **Cultural and Historical bias lead to sub-optimal decisions.**
   - Large social population actions form the basis for these biases and the effects on a person's cognitive information processing function, f_n, are statistical in nature.

Information Bounds in the Board Context

- $H(X_n)$ is the average information shared by a single board member or SME as defined in Equation 2.
- $H(Y_n)$ is the average information received by a single board member or SME also following the definition in Equation 2.
- $H(X_n, Y_n)$ is the joint probability that what was shared by one member and heard by another (the average uncertainty in the total transmission through the board channel).
- $H(Y_n|X_n)$ is the probability that one member actually heard what was stated by another. This brings in the effects of noise (and misunderstanding) in the channel. This focus is on the receiver of the information.
- $H(X_n|Y_n)$ is the equivocation probability that one member actually stated what was heard by another. This brings in the effects of recovery (or proper understanding) of the information sent and is a measure of how well the information is understood by the receiving member.
- Clear Understanding
  - $I(X;Y) = H(X,Y) = H(X) = H(Y)$
  - $I(X;Y) = f(X) = f(Y)$ (Set Theory)
- Total Confusion
  - $I(X;Y) = f(X \cap Y) = 0$

Shared Understanding

- $H(S,D,X,Y,Z) \leq H(S) + H(D) + H(X) + H(Y) + H(Z)$
- $I(X;Y) = f(X \cap Y)$, expected value of mutual information shared in the discussion.
- $H(X,Y) = f(X \cup Y)$, average uncertainty of the discussion.
- $H(X|Y) = f(XY')$,information received by X given the information that Y shared.
- $H(Y|X) = f(YX')$, information shared by Y given the information that X heard.



- **that I(S;D,X|Y,Z) is the information shared between S, D, X, and not Y and Z**
- **Fully informed decisions are contained in the center most ellipsoid, I(S;D,X,Y,Z)**

Board Information Capacity

- **Channel capacity (i.e., board capacity) in information theory is:**

- $C = max\big(I(X;Y)\big) = max(f(X \cap Y))$**.**

- **Thus, channel capacity (i.e., the board capacity) for a decision is defined by the mutual information, or the intersection of information, shared in the board discussion, i.e., , I(S;D,X,Y,Z).**

- Channel capacity is reduced if expertise is missing on the board
  - There is a void in the intersection of understanding (information not fully understood).

  - Note, expertise is missing.
    - Increasing the board size with overlapping expertise makes the board less effective. Can lead to disjoint understanding between the overlapping board members.
    - Knowing and assigning the right expertise to the board for the decision scope chartered is essential to efficient board functioning.

Summary

- **Information Theory provides the mathematical understanding of information flow through an organization**
- **Information Theory can be used to model decision boards and multiple board structures**
- **Control Theory and Cognitive Theory provide the basic information flow structures that Information Theory operates on**
- **Set Theory formulation of Information Theory provides a mathematical model of board understanding**
- **Board Information Flow Capacity is defined by Information Theory**